Capacity Bounds for Two-Hop Interference Networks

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***Abstract*— This paper considers a two-hop interference network, where two users transmit independent messages to their respective receivers with the help of two relay nodes. The transmitters do *not* have direct links to the receivers; instead, two relay nodes serve as intermediaries between the transmitters and receivers. Each hop, one from the transmitters to the relays and the other from the relays to the receivers, is modeled as a Gaussian interference channel, thus the network is essentially a cascade of two interference channels. For this network, achievable symmetric rates for different parameter regimes under decode-and-forward relaying and amplify-and-forward relaying are proposed and the corresponding coding schemes are carefully studied. Numerical results are also provided.**

I. Introduction

The wireless mesh networks are being extensively studied recently due to their potential to improve the performance and throughput of the cellular networks by borrowing the features of ad-hoc networks [1]. The two-hop interference network was recently proposed to model the mesh network from an information theoretic perspective [2]. The model is in essence a cascade of two interference channels: the transmitters communicate to two relay nodes through an interference channel and the two relay nodes communicate to the two receivers through another interference channel.

In [2], the authors studied the achievable region for the model where the relays apply decode-and-forward scheme. For the interference channel in the first hop, since the messages of the two users are independent, the largest achievable region to date was proposed by Han and Kobayashi [3]. The basic idea is for each user to split their message into two parts: the private message, which is only to be decoded by the intended receiver, and the common message, which is to be decoded by both receivers. Although the unintended user's common message is discarded by the receivers in the classic interference channel model, [2] made use of this common message at the two relay nodes as knowledge of them can help boost the rate in the second hop through cooperative transmission. In [2], the authors proposed the superposition coding scheme for each relay node to transmit not only the intended user's private and common messages but also the other user's common message, in order to obtain the coherent combining gain of the common message at the intended receiver.

[4] also considered the two-hop interference network model. Instead of considering the end-to-end transmission rate, the authors focused on the the second hop and explored the possibilities for the two relays to utilize the common message from the unintended user and proposed multiple transmission schemes, such as MIMO broadcast strategy, dirty paper coding, beamforming, and further rate splitting.

However, both [2] and [4] only considered decode-and-forward relaying and focused on the weak interference case for both hops, i.e., the interference link gain is less than the direct link gain. In this paper, we study the model under various parameter regimes using decode-and-forward relaying as well as amplify-and-forward relaying. [2] and [4] also suggested that, if the interference channel in the first hop has strong interference (interference link gain greater than direct link gain), by the standard results for the classic interference channel, it is optimal for the two relays to decode both users messages. Contrary to this, we will show in later sections that this approach can be easily outperformed by switching the roles of the two relays which essentially converts the strong interference channels to weak interference channels. For amplify and forward, we demonstrate that the end-to-end rate may exceed the naive use of cut-set bound which applies to only the decode and forward approach.

The rest of the paper is organized as follows. In section II, we introduce the model for the two-hop interference network. In section III, we focus on the end-to-end transmission rate and analyze the decode-and-forward relaying scheme for the network under different parameter regimes. In section IV, we analyze the amplify-and-forward scheme under various parameter regimes. Section V provides numerical examples to compare various proposed coding schemes. Concluding remarks are given in section VI.

II. Channel Model

The standard two-hop interference network is a cascade of two interference channels with direct transmission link coefficient equal to 1, as shown in Fig. 1.

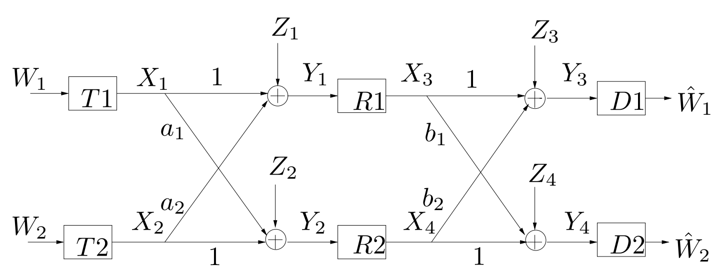


Fig. 1. Two-hop interference network in standard form

Transmitter 1 (T_1) has message $W_1 \in \{1, 2, \cdots, 2^{nR_1}\}$ to be transmitted to destination D_1 and transmitter 2 (T_2) has message $W_2 \in \{1, 2, \cdots, 2^{nR_2}\}$ to be transmitted to destination D_2. a_1, a_2, b_1 and b_2 are fixed positive numbers, Z_1, Z_2, Z_3 and Z_4 are independent Gaussian distributed variables with zero mean and unit variance. The average power constraints

for the input signals X_1, X_2, X_3 and X_4 are P_{11}, P_{12}, P_{21} and P_{22}, respectively.

In order to simplify the analysis of this complicated channel model and better compare our results with the existing ones, we follow the convention of [2] and [4] by only considering the symmetric interference channels, i.e.,

$$a_1 = a_2 \triangleq a \quad (1)$$
$$b_1 = b_2 \triangleq b \quad (2)$$
$$P_{11} = P_{12} \triangleq P_1 \quad (3)$$
$$P_{21} = P_{22} \triangleq P_2 \quad (4)$$

In addition, we focus primarily on the symmetric rate, i.e., the case with $R_1 = R_2$.

III. Decode and Forward

In this section, we propose capacity bounds for the two-hop interference network in various parameter regimes using decode-and-forward relaying. Under the full duplex condition, the transmission is conducted across a large number of blocks. In each block, the relays receive the new messages of the current block from the transmitters, and transmit the information of the previous block to the desitnation. We assume the number of blocks is large enough to ignore the penalty incurred in the first and the last blocks.

A. $0 < a < 1, 0 < b < 1$

In [2], the authors proposed achievable transmission rates for the case that both hops have weak interference, i.e., $a < 1$ and $b < 1$. Specifically, they applied Han-Kobayashi's scheme to the first hop by splitting each user's message into two parts, namely, W_1 into private message $W_{1p} \in \{1, \cdots, 2^{nR_{1p}}\}$ and common message $W_{1c} \in \{1, \cdots, 2^{nR_{1c}}\}$ and W_2 into private message $W_{2p} \in \{1, \cdots, 2^{nR_{2p}}\}$ and common message $W_{2c} \in \{1, \cdots, 2^{nR_{2c}}\}$. Each relay not only decodes the private and common messages from the intended user, but also decodes the common message from the other user. Since the Han-Kobayashi region is based on simultaneous decoding of the three messages (1 private message and 2 common messages), which is very complicated to compute, [4] simplified it by proposing sequential decoding: each relay first decodes the two common messages, subtract them out, then decode the private message. By restricting the analysis to the symmetric rate [4], i.e., $R_{1p} = R_{2p} = R_p^{(1)}$, $R_{1c} = R_{2c} = R_c^{(1)}$, we have achievable rates in the first hop

$$R_p^{(1)} = \gamma\left(\frac{\alpha P_1}{1 + a^2 \alpha P_1}\right) \quad (5)$$
$$R_c^{(1)} = \min\left\{\gamma\left(\frac{a^2 \bar{\alpha} P_1}{\sigma_1^2}\right), \frac{1}{2}\gamma\left(\frac{(1+a^2)\bar{\alpha} P_1}{\sigma_1^2}\right)\right\} \quad (6)$$

where αP_1 is the power allocated to the private message and $\bar{\alpha} P_1 = (1 - \alpha) P_1$ is the power allocated to the common message. $\sigma_1^2 = 1 + (1+a^2)\alpha P_1$. $\gamma(x)$ is defined as $\frac{1}{2}\log(1+x)$. The superscript "(1)" denotes the first hop. (6) is from the capacity region of the MAC channel consisting of the two common messages, treating the private messages as noise; (5) is the decoding of the private message treating the other private message as noise.

For the second hop, [2] proposed superposition scheme at the two relays such that coherent combining can be achieved at the destinations. This scheme was outperformed by the dirty paper coding (DPC) scheme proposed in [4] for the very weak interference case, i.e., when b is very small. The idea is for the two relays to encode one of the common messages using DPC, thus treating the other common message as known interference. Therefore, this known interference will not affect the unintended destination. However, due to the nonlinearity of the DPC, the dirty paper decoded common message cannot be subtracted out. Thus, [4] also suggested to dirty paper code the private message treating both common messages as known interference. Besides, the common message that is treated as known interference is decoded at its intended destination by treating the other common message (dirty paper coded) as well as the two private messages as noise. Since either common message can be dirty paper coded against the other common message, there are two transmission modes and one should time share between them to maximize the sum rate [4]. Again, by only considering the symmetric rates, the achievable rates under the DPC scheme for the second hop are [4]

$$R_{p,DPC}^{(2)} = \gamma\left(\frac{\beta P_2}{1 + b^2 \beta P_2}\right) \quad (7)$$
$$R_{c,DPC}^{(2)} = \frac{1}{2}\gamma\left(\frac{(1-b^2)^2 \bar{\beta}^2 P_2^2}{\sigma_2^4} + \frac{2(1+b^2)\bar{\beta} P_2}{\sigma_2^2}\right) \quad (8)$$

where βP_2 is the power allocated to the private message, $\sigma_2^2 = 1 + (1+b^2)\beta P_2$ since the private messages from both users are treated as noise when decoding common messages. (7) is decoding the private message treating the other user's private message as noise, since the effect of the two common messages disappears due to the DPC; (8) is from the optimization problem which maximizes the sum rate of the two common messages.

In the DPC scheme, the common message that is treated as known interference is decoded by its intended receiver treating the other user's common message and private messages as noise. However, when the interference link of the second hop gets stronger, i.e., b gets larger, the interference incurred by the common message and private message from the other user may be too strong to be treated as noise. Therefore, it may be beneficial for the receivers to decode the common message and even the private message from the other user, like in the strong interference channel, whose capacity is that of the compound MAC. To make the coding scheme more general, we do not let the receivers decode all the private messages. Instead, we further split the private message W_{1p} from the first hop into two parts, $W_{1pp} \in \{1, 2, \cdots, 2^{nR_{1pp}}\}$ and $W_{1pc} \in \{1, 2, \cdots, 2^{nR_{1pc}}\}$, where W_{1pp} is the sub-private message only decoded at the intended receiver, and W_{1pc} is the sub-common message decoded at both receivers. The private message W_{2p} is split in the same fashion into W_{2pp} and W_{2pc}. There are five messages (two common messages, two sub-common messages and one sub-private message) to be decoded by each

receiver, which yields very complex expression for the rate region if we use simultaneous decoding. Instead, we will adopt sequential decoding and fix the decoding order as follows: first, simultaneously decode the two common messages W_{1c} and W_{2c}, subtract them out; second, simultaneously decode the two sub-common messages W_{1pc} and W_{2pc}, subtract them out; third, decode the sub-private message W_{1pp} by receiver 1 (or W_{2pp} by receiver 2). Consequently, the symmetric achievable rate region is

$$R_c \leq \gamma\left(\frac{(\sqrt{P_{c1}}+b\sqrt{P_{c2}})^2}{1+(1+b^2)P_p}\right) \tag{9}$$
$$R_c \leq \gamma\left(\frac{(\sqrt{P_{c2}}+b\sqrt{P_{c1}})^2}{1+(1+b^2)P_p}\right) \tag{10}$$
$$2R_c \leq \gamma\left(\frac{(\sqrt{P_{c1}}+b\sqrt{P_{c2}})^2+(\sqrt{P_{c2}}+b\sqrt{P_{c1}})^2}{1+(1+b^2)P_p}\right) \tag{11}$$
$$R_{pc} \leq \gamma\left(\frac{P_{pc}}{1+(1+b^2)P_{pp}}\right) \tag{12}$$
$$R_{pc} \leq \gamma\left(\frac{b^2P_{pc}}{1+(1+b^2)P_{pp}}\right) \tag{13}$$
$$2R_{pc} \leq \gamma\left(\frac{(1+b^2)P_{pc}}{1+(1+b^2)P_{pp}}\right) \tag{14}$$
$$R_{pp} \leq \gamma\left(\frac{P_{pp}}{1+b^2P_{pp}}\right) \tag{15}$$

where power P_p is allocated to the private message, P_{c1} is allocated to the intended common message, P_{c2} is allocated to the interfering common message, and $P_p+P_{c1}+P_{c2}=P_2$. Also, P_{pc} is for the sub-common message and P_{pp} is for the sub-private message and $P_{pc}+P_{pp}=P_p$. If we fix P_p and maximize R_c under $P_{c1}+P_{c2}\leq P_2-P_p$, the optimal $R_c^*=\frac{1}{2}\gamma\left(\frac{(1+b)^2(P_2-P_p)}{1+(1+b^2)P_p}\right)$ is achieved when $P_{c1}=P_{c2}=\frac{1}{2}(P_2-P_p)$ [4]. Therefore, the symmetric rates for the second hop under the MAC scheme is

$$R_{p,MAC}^{(2)} = \max_{\alpha}\left\{\min\left[\gamma\left(\frac{b^2\bar{\alpha}\beta P_2}{\sigma_3^2}\right),\frac{1}{2}\gamma\left(\frac{(1+b^2)\bar{\alpha}\beta P_2}{\sigma_3^2}\right)\right] + \gamma\left(\frac{\alpha\beta P_2}{1+b^2\alpha\beta P_2}\right)\right\} \tag{16}$$
$$R_{c,MAC}^{(2)} = \frac{1}{2}\gamma\left(\frac{(1+b)^2\bar{\beta}P_2}{1+(1+b^2)\beta P_2}\right) \tag{17}$$

where $\sigma_3^2=1+(1+b^2)\alpha\beta P_2$ and $\alpha,\beta\in[0,1]$.

This scheme is more general than the cooperative transmission scheme in [2] in that we further split the first hop's private messages into two parts in the second hop. This scheme is similar to the "layered coding with beamforming" scheme in [4], with the difference that we only consider the coherent beamforming here and disregard the zero forcing beamforming scheme which proves to be always worse than the DPC scheme.

Theorem 1: The achievable symmetric rate ($R_1=R_2=R$) for the symmetric interference network is the solution to the following optimization problem:

$$R = \max_{\alpha,\beta\in[0,1]} R_p+R_c \tag{18}$$
$$\text{s.t.}(R_p,R_c)\in\mathcal{R}(R_p^{(1)},R_c^{(1)})\cap\mathcal{R}(R_p^{(2)},R_c^{(2)}) \tag{19}$$

where $R_p^{(1)}$ and $R_c^{(1)}$ are given in (5)-(6). $\mathcal{R}(R_p^{(2)},R_c^{(2)})$ is defined as the convex closure of the union of $\mathcal{R}(R_{p,DPC}^{(2)},R_{c,DPC}^{(2)})$ and $\mathcal{R}(R_{p,MAC}^{(2)},R_{c,MAC}^{(2)})$, where $R_{p,DPC}^{(2)}$ and $R_{c,DPC}^{(2)}$ are given in (7)-(8), and $R_{p,MAC}^{(2)}$ and $R_{c,MAC}^{(2)}$ are given in (16)-(17).

B. $a>1, b>1$

If the first hop has strong interference, i.e., $a>1$, both [2] and [4] let both relays decode both users' messages in the first hop, as this is the optimal scheme for interference channels with strong interference. Using this scheme, for the symmetric rates ($R_1=R_2=R^{(1)}$), we have

$$R^{(1)} \leq \gamma(P_1) \tag{20}$$
$$R^{(1)} \leq \gamma(a^2P_1) \tag{21}$$
$$R^{(1)}+R^{(1)} \leq \gamma(P_1+a^2P_1) \tag{22}$$

Thus,

$$R^{(1)}=\min\left(\gamma(P_1),\frac{1}{2}\gamma((1+a^2)P_1)\right) \tag{23}$$

In other words, for very strong interference case $a^2\geq 1+P_1$, $R^{(1)}=\gamma(P_1)$; for $1<a^2<1+P_1$, $R^{(1)}=\frac{1}{2}\gamma((1+a^2)P_1)$.

After the first hop, since both relays have knowledge of both users' messages, the second hop reduces to the Gaussian vector broadcast channel with per antenna power constraint, for which we know the DPC scheme is optimal. By time sharing between the two DPC modes and maximizing the sum rate, we obtain the achievable symmetric rate for the second hop

$$R^{(2)}=\frac{1}{2}\gamma((b^2-1)^2P_2^2+2P_2(1+b^2)). \tag{24}$$

Therefore the achievable rate for the entire network is

$$R=\min\{R^{(1)},R^{(2)}\}. \tag{25}$$

The above analysis seems to be a natural way to deal with the strong interference case, and for each hop, the transmission scheme is optimal. However, optimality in each hop does not guarantee optimality of the entire network. Indeed, for the entire system, the combination of the two optimal schemes is no longer optimal. An easy way to outperform the above scheme is to switch the role of the two relays. Specifically, we make relay R_2 as the "intended" relay for the first user T_1, and relay R_1 as the intended relay for the second user T_2. In this way, the first hop is converted into an interference channel with weak interference. Consequently, the second hop is converted into another weak interference channel as shown

in Fig.2. After some simple scaling, this two-hop network becomes

$$Y_1' = X_1 + \frac{1}{a}X_2 + Z_1' \quad (26)$$

$$Y_2' = \frac{1}{a}X_1 + X_2 + Z_2' \quad (27)$$

$$Y_3' = X_3 + \frac{1}{b}X_4 + Z_3' \quad (28)$$

$$Y_4' = \frac{1}{b}X_3 + X_4 + Z_4' \quad (29)$$

where $Z_1', Z_2' \sim N(0, 1/a^2)$, $Z_3', Z_4' \sim N(0, 1/b^2)$ are independent.

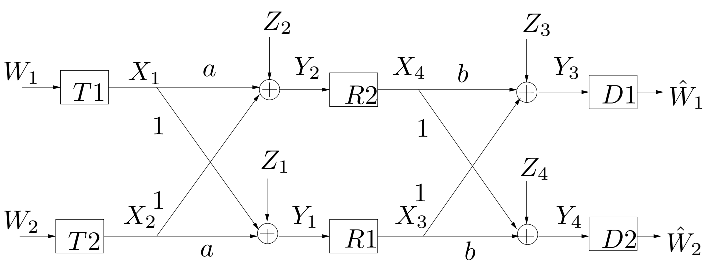


Fig. 2. Two-hop interference network transformation

Therefore, this strong interference two-hop network reduces to case III-A where both hops are weak interference channels. Using Han-Kobayashi scheme in the first hop and combining DPC and MAC in the second hop, and going through the same derivation, we obtain the symmetric rates in the first hop

$$R_p^{(1)} = \gamma\left(\frac{a^2\alpha P_1}{1+\alpha P_1}\right) \quad (30)$$

$$R_c^{(1)} = \min\left\{\gamma(\frac{\bar{\alpha}P_1}{\sigma_1^2}), \frac{1}{2}\gamma\left(\frac{(1+a^2)\bar{\alpha}P_1}{\sigma_1^2}\right)\right\} \quad (31)$$

where $\alpha \in [0,1]$ and $\sigma_1^2 = 1 + (1+a^2)\alpha P_1$.

The symmetric rates in the second hop under DPC is

$$R_{p,DPC}^{(2)} = \gamma\left(\frac{b^2\beta P_2}{1+\beta P_2}\right) \quad (32)$$

$$R_{c,DPC}^{(2)} = \frac{1}{2}\gamma\left(\frac{(b^2-1)^2\bar{\beta}^2P_2^2}{\sigma_2^4} + \frac{2(1+b^2)\bar{\beta}P_2}{\sigma_2^2}\right) \quad (33)$$

where $\beta \in [0,1]$ and $\sigma_2^2 = 1 + (1+b^2)\beta P_2$.

The symmetric rates in the second hop under MAC is

$$R_{p,MAC}^{(2)} = \max_{\alpha}\left\{\min\left[\gamma\left(\frac{\bar{\alpha}\beta P_2}{\sigma_3^2}\right), \frac{1}{2}\gamma\left(\frac{(1+b^2)\bar{\alpha}\beta P_2}{\sigma_3^2}\right)\right] + \gamma\left(\frac{b^2\alpha\beta P_2}{1+\alpha\beta P_2}\right)\right\} \quad (34)$$

$$R_{c,MAC}^{(2)} = \frac{1}{2}\gamma\left(\frac{(1+b)^2\bar{\beta}P_2}{1+(1+b^2)\beta P_2}\right) \quad (35)$$

where $\sigma_3^2 = 1 + (1+b^2)\alpha\beta P_2$ and $\alpha, \beta \in [0,1]$.

Theorem 2: The solution to the following optimization problem is achievable for the two-hop network when $a > 1$ and $b > 1$:

$$R = \max_{\alpha,\beta\in[0,1]} R_p + R_c \quad (36)$$

$$\text{s.t.} (R_p, R_c) \in \mathcal{R}(R_p^{(1)}, R_c^{(1)}) \cap \mathcal{R}(R_p^{(2)}, R_c^{(2)}) \quad (37)$$

where $R_p^{(1)}$ and $R_c^{(1)}$ are given in (30)-(31). $\mathcal{R}(R_p^{(2)}, R_c^{(2)})$ is defined as the convex closure of the union of $\mathcal{R}(R_{p,DPC}^{(2)}, R_{c,DPC}^{(2)})$ and $\mathcal{R}(R_{p,MAC}^{(2)}, R_{c,MAC}^{(2)})$, where $R_{p,DPC}^{(2)}$ and $R_{c,DPC}^{(2)}$ are given in (32)-(33), and $R_{p,MAC}^{(2)}$ and $R_{c,MAC}^{(2)}$ are given in (34)-(35).

Note that when $\alpha = \beta = 0$, let $\mathcal{R}(R_p^{(2)}, R_c^{(2)}) = \mathcal{R}(R_{p,DPC}^{(2)}, R_{c,DPC}^{(2)})$, the rate R defined in (36) reduces to that of (25). Since $\mathcal{R}(R_p^{(2)}, R_c^{(2)})$ is always a superset of $\mathcal{R}(R_{p,DPC}^{(2)}, R_{c,DPC}^{(2)})$, the achievable rate (25) is always a subset of (36).

C. $0 < a < 1, b > 1$

For the first hop, it is a weak interference channel, the transmission strategy is the same as case III-A: the Han-Kobayashi scheme. Thus, the symmetric achievable rate is $(R_p^{(1)}, R_c^{(1)})$ given in (5)-(6).

For the second hop, we can still use DPC scheme, thus yielding rates $(R_{p,DPC}^{(2)}, R_{c,DPC}^{(2)})$ given in (7)-(8). Now consider the MAC scheme. From the standard result of strong interference channel, the capacity is achieved when both user's messages are decoded by both receivers, as in the case of compound MAC. Thus, for the MAC scheme proposed in section III-A, we should modify it by letting both receivers decode all the messages, both private and common, instead of further splitting the private message. As such, we should set $\alpha = 0$ in (16)-(17). Also notice that $b > 1$, the symmetric achievable rates for the MAC scheme become

$$R_{p,MAC}^{(2)} = \min\left\{\gamma(\beta P_2), \frac{1}{2}\gamma((1+b^2)\beta P_2)\right\} \quad (38)$$

$$R_{c,MAC}^{(2)} = \frac{1}{2}\gamma\left(\frac{(1+b)^2\bar{\beta}P_2}{1+(1+b^2)\beta P_2}\right) \quad (39)$$

Therefore, for the case $0 < a < 1, b > 1$, the symmetric achievable rate for the two hop network has the same form of that in Theorem 1, except that $R_{p,MAC}^{(2)}$ and $R_{c,MAC}^{(2)}$ are given in (38)-(39).

D. $a > 1, 0 < b < 1$

If we stick to the roles of the two relays, for the first hop, the two relays should decode both users' messages; for the second hop, we apply DPC scheme for the weak interference channel. However, similar to case III-B, it can be verified that this scheme is easily outperformed if we switch the role of the two relays. Consequently, the first hop becomes a weak interference channel and the second hop becomes a strong interference channel. We can directly apply the results from case III-C, with only minor modifications: change the channel gains a and b into $\frac{1}{a}$ and $\frac{1}{b}$ respectively, and change the variance of noise Z_1 and Z_2 to $\frac{1}{a^2}$, and change the variance of noise Z_3 and Z_4 to $\frac{1}{b^2}$. Thus, the total symmetric rate of the

two hop network becomes the same form of that in Theorem 2 except that $R_{p,MAC}^{(2)}$ and $R_{c,MAC}^{(2)}$ are given in (40)-(41).

$$R_{p,MAC}^{(2)} = \min\left\{\gamma(b^2\beta P_2), \frac{1}{2}\gamma((1+b^2)\beta P_2)\right\} \quad (40)$$

$$R_{c,MAC}^{(2)} = \frac{1}{2}\gamma\left(\frac{(1+b)^2\bar{\beta}P_2}{1+(1+b^2)\beta P_2}\right) \quad (41)$$

For the second hop, the DPC scheme and the MAC scheme are both needed for all the parameter regimes. Neither scheme can dominate the other.

From the previous analysis of the four parameter regimes, we have the following theorem.

Theorem 3: For the two hop interference network with the transmission scheme of decode and forward relaying, if the first hop has weak interference, one should apply the HK scheme directly; if the first hop has strong interference, it is always favorable to convert it into a weak interference channel by switching the roles of the two relays, as in Fig. 2, and then apply the HK scheme. In other words, with strong interference in the first hop, rate splitting after role switching of the two relays can always achieve a rate region no smaller than that achieved by both relays decoding all the messages without role switching.

Proof: If the two relays do not switch roles, for strong interference in the first hop, the optimal scheme is for both the two relays to decode all the messages of the two users. Then, the optimal scheme for the second hop is to use DPC scheme as in the MIMO broadcast channel. However, these schemes are special cases of the transmission schemes if we switch the roles of the two relays and apply the HK scheme to the first hop(simply by allocating zero power to the private messages after rate splitting). Therefore, role exchange for the two relay nodes is always preferred for strong interference in the first hop. ■

E. Half Duplex

If the transmission is conducted in the half duplex fashion, the two relays cannot receive and transmit at the same time. In this case, the transmission in the two hops cannot proceed simultaneously. When transmitting in the first hop, the relays are in the listening mode and the two users T_1, T_2 transmit their messages with N_1 channel uses to the relays. In the second hop, after decoding the received messages, the two relays R_1, R_2 transmit with N_2 channel uses to the two destinations D_1, D_2. Thus, the transmission schemes discussed for the full duplex case can be directly applied to the half duplex case, only with the overall rates reduced due to the extra channel uses needed.

Following the schemes proposed for the full duplex mode, we always do rate splitting and transmit private as well as common messages in the first hop. Thus, both private and common messages should be successfully delivered to the destinations in the second hop, which yields:

$$R_p^{(1)}N_1 \le R_p^{(2)}N_2 \quad (42)$$

$$R_c^{(1)}N_1 \le R_c^{(2)}N_2 \quad (43)$$

The minimum channel uses needed in the second hop is

$$N_2 = N_1 \cdot \max\left(\frac{R_p^{(1)}}{R_p^{(2)}}, \frac{R_c^{(1)}}{R_c^{(2)}}\right) \quad (44)$$

Therefore, the overall rate achieved for the entire system is

$$R = \frac{(R_p^{(1)}+R_c^{(1)})N_1}{N_1+N_2} = \frac{R_p^{(1)}+R_c^{(1)}}{1+\max\left(\frac{R_p^{(1)}}{R_p^{(2)}}, \frac{R_c^{(1)}}{R_c^{(2)}}\right)} \quad (45)$$

Theorem 4: $R^* = \max R$ is the achievable symmetric rate ($R_1 = R_2 = R^*$) in the half duplex two-hop interference network, where R is defined in (45).

IV. Amplify and Forward

In this section, we focus on the transmission rates achieved by amplify and forward relaying. We show that this scheme can outperform decode and forward relaying under certain conditions.

For amplify and forward relaying, we still focus on the symmetric channel model as defined in (1)-(4).

A. In-phase Relaying

We first analyze the achievable rates for the so-called in-phase transmission, where the two relays simply scale their received signals with the same polarity. This is the usual amplify and forward scheme and we emphasize in-phase here to contrast with the out-of-phase approach described later. In the first hop, the received signals at the relays are

$$Y_1 = X_1 + aX_2 + Z_1 \quad (46)$$

$$Y_2 = aX_1 + X_2 + Z_2 \quad (47)$$

If they use the full power for amplifying in the second hop, we have

$$X_3 = cY_1 \quad (48)$$

$$X_4 = cY_2 \quad (49)$$

where $c = \sqrt{\frac{P_2}{(1+a^2)P_1+1}}$. Therefore

$$Y_3 = cY_1 + bcY_2 + Z_3 \quad (50)$$

$$Y_4 = bcY_1 + cY_2 + Z_4 \quad (51)$$

after scaling,

$$Y_3' = (1+ab)X_1 + (a+b)X_2 + Z_1 + bZ_2 + Z_3/c \quad (52)$$

$$Y_4' = (a+b)X_1 + (1+ab)X_2 + bZ_1 + Z_2 + Z_4/c \quad (53)$$

Due to the fact that receivers D_1 and D_2 do not talk to each other, we can modify the model in (52)-(53) to the following one without affecting its capacity region:

$$Y_3 = (1+ab)X_1 + (a+b)X_2 + Z_3' \quad (54)$$

$$Y_4 = (a+b)X_1 + (1+ab)X_2 + Z_4' \quad (55)$$

where $Z_3, Z_4 \sim N(0, 1+b^2+1/c^2)$ are independent variables.

1) Strong Interference: It is clear that the model in (54)-(55) will be a strong interference channel if $a+b>1+ab$, i.e.,

$$\{a<1,b>1\} \quad \text{or} \quad \{a>1,b<1\} \tag{56}$$

For this model, the optimal scheme is for the two receivers to decode both users messages, and the capacity region is known as

$$R_1 \leq \gamma\left(\frac{(1+ab)^2P_1}{1+b^2+1/c^2}\right) \tag{57}$$

$$R_2 \leq \gamma\left(\frac{(1+ab)^2P_1}{1+b^2+1/c^2}\right) \tag{58}$$

$$R_1+R_2 \leq \gamma\left(\frac{((1+ab)^2+(a+b)^2)P_1}{1+b^2+1/c^2}\right) \tag{59}$$

Thus, the symmetric achievable rate $(R_1=R_2=R)$ is

$$R=\min\left\{\gamma\left(\frac{(1+ab)^2P_1}{1+b^2+1/c^2}\right), \frac{1}{2}\gamma\left(\frac{((1+ab)^2+(a+b)^2)P_1}{1+b^2+1/c^2}\right)\right\} \tag{60}$$

2) Weak Interference: On the other hand, if $a+b<1+ab$, i.e.,

$$\{a>1,b>1\} \quad \text{or} \quad \{a<1,b<1\} \tag{61}$$

the model (54)-(55) becomes a weak interference channel, for which the Han-Kobayashi's scheme is the best known scheme. Similar to the analysis in III-A, the symmetric private rate and common rate are

$$R_p \leq \gamma\left(\frac{(1+ab)^2\alpha P_1}{(a+b)^2\alpha P_1+b^2+1+1/c^2}\right) \tag{62}$$

$$R_c \leq \min\left\{\gamma\left(\frac{(a+b)^2\bar{\alpha}P_1}{\sigma_1^2}\right), \frac{1}{2}\gamma\left(\frac{\sigma_2^2}{\sigma_1^2}\right)\right\} \tag{63}$$

where $\sigma_1^2=((1+ab)^2+(a+b)^2)\alpha P_1+b^2+1+1/c^2$ and $\sigma_2^2=((1+ab)^2+(a+b)^2)\bar{\alpha}P_1$. The symmetric rate for the whole system is

$$R=\max_{\alpha\in[0,1]} R_p+R_c. \tag{64}$$

It is interesting to note that for the method of amplify and forward relaying, the analysis also shows the four parameter regimes can actually be divided into two categories, in the sense of transmission and decoding schemes, where $(a<1,b<1)$ and $(a>1,b>1)$ belong to one category, and $(a<1,b>1)$ and $(a>1,b<1)$ belong to the other category. This coincides with the analysis of the decode and forward relaying in the previous section.

B. Out-of-phase Relaying

Besides in-phase relaying, the two relays can also purposely make the relayed signal out of phase by exactly 180^o, i.e., change the sign of the relay output. We show in this subsection that this scheme can have very nice performance under certain conditions.

Again, by using full power at the two relays and making the relayed signals out of phase by 180^o, we have

$$X_3 = -cY_1 \tag{65}$$

$$X_4 = cY_2 \tag{66}$$

where $c=\sqrt{\frac{P_2}{(1+a^2)P_1+1}}$. Therefore,

$$Y_3 = -cY_1+bcY_2+Z_3 \tag{67}$$

$$Y_4 = -bcY_1+cY_2+Z_4 \tag{68}$$

which, after scaling, is

$$Y_3^{'} = (ab-1)X_1+(b-a)X_2-Z_1+bZ_2+Z_3/c \tag{69}$$

$$Y_4^{'} = (a-b)X_1-(ab-1)X_2-bZ_1+Z_2+Z_4/c \tag{70}$$

Since D_1 and D_2 cannot talk to each other, we can modify the model (69)-(70) to the following model with the same capacity region:

$$Y_3 = (ab-1)X_1+(b-a)X_2+Z_3^{'} \tag{71}$$

$$Y_4 = (a-b)X_1+(1-ab)X_2+Z_4^{'} \tag{72}$$

where $Z_3, Z_4 \sim N(0, 1+b^2+1/c^2)$ are independent random noises.

1) Strong Interference: For model (71)-(72), this becomes a strong interference channel if $|ab-1|<|b-a|$, i.e.,

$$\{a<1,b>1\} \quad \text{or} \quad \{a>1,b<1\}. \tag{73}$$

This is exactly the same condition as the strong interference case in section IV-A. Similar to the analysis of IV-A, we can express the symmetric rate for the strong interference case as

$$R=\min\left\{\gamma\left(\frac{(1-ab)^2P_1}{1+b^2+1/c^2}\right), \frac{1}{2}\gamma\left(\frac{((1-ab)^2+(a-b)^2)P_1}{1+b^2+1/c^2}\right)\right\}. \tag{74}$$

Obviously, the rate in (74) is less than that in (60). Thus, for amplify and forward relaying, under condition (73), we should employ in-phase relaying at the two relays.

2) Weak Interference: When $|ab-1|>|b-a|$, the model (71)-(72) becomes a weak interference channel, i.e.,

$$\{a>1,b>1\} \quad \text{or} \quad \{a<1,b<1\} \tag{75}$$

which is also consistent with the condition of the weak interference case in section IV-A. Using Han-Kobayashi's scheme, we get the symmetric private rate and common rate

$$R_p \leq \gamma\left(\frac{(1-ab)^2\alpha P_1}{(a-b)^2\alpha P_1+b^2+1+1/c^2}\right) \tag{76}$$

$$R_c \leq \min\left\{\gamma\left(\frac{(1-ab)^2\bar{\alpha}P_1}{\sigma_1^2}\right), \frac{1}{2}\gamma\left(\frac{\sigma_2^2}{\sigma_1^2}\right)\right\} \tag{77}$$

where $\sigma_1^2=((1-ab)^2+(a-b)^2)\alpha P_1+b^2+1+1/c^2$ and $\sigma_2^2=((1-ab)^2+(a-b)^2)\bar{\alpha}P_1$.

Comparing rates of (76)-(77) and that of (62)-(63), it can be easily verified that when $a=b$ and $ab>>1$(or $ab<<1$), (76)-(77) will outperform (62)-(63).

If we consider this two hop interference channel network as two water pipes cascaded with each hop as one pipe, it is very nature to think the total throughput of the entire system should

be bounded by the capacities of both pipes (e.g., min cut), which is exactly the case for the decode and forward relaying. However, for the amplify and forward relaying, we show that this natural analogy is not valid, i.e., the total throughput can be larger than the capacities of both "pipes".

If $a = b$, the model (71)-(72) becomes two parallel AWGN channels and the rates for both channels are the same:

$$R = \gamma\left(\frac{(1-a^2)^2 P_1}{1+a^2+1/c^2}\right) = \gamma\left(\frac{(1-a^2)^2 P_1 P_2}{(1+a^2)(P_1+P_2)+1}\right) \quad (78)$$

If $a = b > 1$, according to Theorem 3, the capacity of each of the two hops is always less than or equal to that of the transformed channel where we switch the roles of the two relays, thus converting the strong interference into weak interference for both hops. Therefore, without loss of generality, we only consider the case when $a = b < 1$. For the interference channel of the first hop, by [5]–[7], the channel has "noisy interference" when

$$a(a^2 P_1 + 1) \leq \frac{1}{2} \text{i.e.,} P_1 \leq \frac{1}{a^2}\left(\frac{1}{2a} - 1\right) \quad (79)$$

Under noisy interference, we know the sum rate capacity of the channel [5]–[7], which is achieved by treating the other user's signal as pure noise. Thus, the corresponding symmetric capacity is

$$C_1 = \gamma\left(\frac{P_1}{1+a^2 P_1}\right) \quad (80)$$

If $P_2 = P_1$, the symmetric capacity of the second hop is also $C_2 = C_1 = \gamma\left(\frac{P_1}{1+a^2P_1}\right)$. In order for the rate (78) to exceed the capacity of both hops for $P_1 = P_2$, i.e.,

$$\gamma\left(\frac{(1-a^2)^2 P_1 P_2}{(1+a^2)(P_1+P_2)+1}\right) > \gamma\left(\frac{P_1}{1+a^2 P_1}\right) \quad (81)$$

we need to satisfy

$$P_1 > \frac{1+4a^2-a^4+\sqrt{(1+4a^2-a^4)^2+4a^2(1-a^2)^2}}{2a^2(1-a^2)^2} \quad (82)$$

Combining (79), we get

$$\frac{1+4a^2-a^4+\sqrt{(1+4a^2-a^4)^2+4a^2(1-a^2)^2}}{2a^2(1-a^2)^2} < P_1 < \frac{1}{a^2}\left(\frac{1}{2a}-1\right) \quad (83)$$

We can easily check that when a is close to 0, the lower bound of (83) is $O(\frac{1}{a^2})$ and the upper bound of (83) is $O(\frac{1}{a^3})$, which indicates that when a is close to 0, such P_1 does exist. For example, when $a = 0.15$, the bound in (83) becomes $51.6 < P_1 < 103.7$.

The above example is for $a = b < 1$. Similarly, for $a = b > 1$, due to the previous analysis that these two cases are essentially identical (by switching the roles of the two relays), it can be verified that when $a = b >> 1$, the transmission rate for the whole system can also exceed the capacity of each individual interference channel. The details are omitted here.

Although the above results are obtained for $a = b$, we comment that even when $a \neq b$ but are close, one can still find parameter regimes for which the out-of-phase scheme is favored, i.e., has a larger symmetric rate.

V. Numerical Examples

For both decode-and-forward relaying and amplify-and-forward relaying, when the the first hop has strong interference, i.e., $a > 1$, it is always preferred to switch the roles of the two relays and convert the channel into a weak interference channel. Without loss of generality, we only focus on the weak interference case of the first hop, i.e., $a < 1$. First, we compare the effect of the two schemes in the second hop, namely DPC scheme and MAC scheme, under different channel parameters for the decode-and-forward relaying.

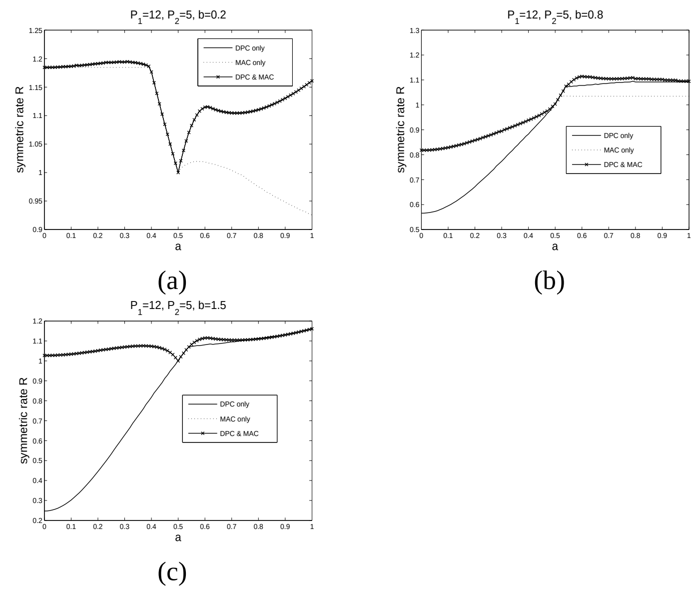
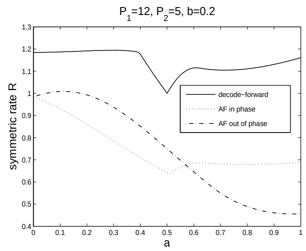


Fig. 3. Comparison of DPC scheme and MAC scheme in the second hop for the decode-and-forward relaying.

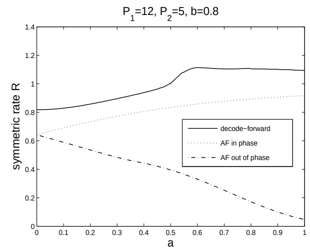
Fig. 3 (a) shows that when the interference gain of the second hop b is very small, the DPC scheme is dominating for $a \in [0, 1]$ and the symmetric rate for combining DPC and MAC will coincide with that of DPC scheme only. The difference between DPC and MAC becomes dramatic when $a > 0.5$. That is because in this regime, the HK scheme will produce significant amount of common information in the first hop, and MAC scheme requires the common information to be decoded by both receivers, which negatively affects the total rate since b is small at the second hop. However, when b gets larger, as shown in (b), MAC scheme will beat DPC for $a < 0.5$ but will be outperformed by DPC for $a > 0.5$. Since for $a < 0.5$, there is significant amount of private messages produced by HK scheme in the first hop, which will be treated as noise in the DPC scheme, but will be partially decoded in the MAC scheme, thus MAC will perform better. However for $a > 0.5$, the common messages from the first hop dominates. Since DPC scheme can cancel the interference effect of other user's common messages, this advantage beats the MAC scheme where the common messages need to be decoded by both receivers when b is not strong enough ($b = 0.8$). Note that the combination of DPC and MAC will outperform both of the individual schemes for $a > 0.5$ because of the time

sharing effect of the two rate regions. When b gets strong enough as in (c), the MAC scheme will far outperform DPC when a is small, but will be close to DPC when a gets larger.

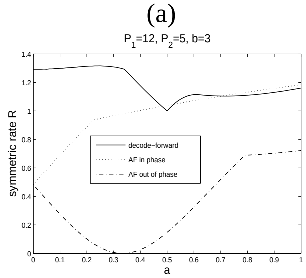
Next, we show in Fig. 4 the comparison of decode-and-forward relaying and amplify-and-forward relaying (both in phase and 180^o out of phase) in low SNR regime.



(a)



(b)



(c)

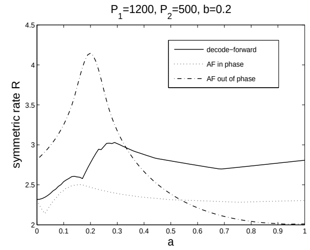
Fig. 4. Comparison of decode-and-forward relaying and amplify-and-forward relaying in low SNR regime

It can be seen that for the low SNR regime, when b is small, the amplify-and-forward relaying scheme (for both in phase and 180^o out of phase) will always be outperformed by the decode-and-forward scheme, as shown in (a) and (b). When b gets strong enough, as shown in (c), the in phase amplify-and-forward relaying may outperform, but not by much, the decode-and-forward scheme when a is close to 1. In other words, for the low SNR regime, decode-and-forward scheme is preferred over amplify-and-forward scheme. However, at high SNR regime, it is a different story.

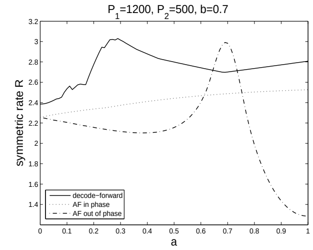
As shown in Fig. 5, in the high SNR regime, when $b < 1$, the performance of amplify-and-forward relaying with 180^o out of phase is the best when a is close to b. This is because when $a = b$, the channel becomes two parallel AWGN channels, which has the best performance under the high SNR regime. However, away from the peak of $a = b$, the amplify-and-forward relaying with 180^o out of phase is still the worst. When $b \geq 1$, since $a \in [0, 1]$, the peak of $a = b$ does not exist any more, thus, the performance of the out of phase amplify-and-forward relaying becomes the worst for all values of a. In this case, the decode-and-forward scheme remains the best of all.

VI. Conclusion

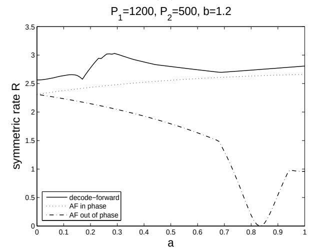
In this paper, we investigated and compared coding schemes for the two hop interference network under various channel parameters regimes. Our analysis shows that if the first hop has strong interference, i.e., $a > 1$, it is always beneficial to switch



(a)



(b)



(c)

Fig. 5. Comparison of decode-and-forward relaying and amplify-and-forward relaying in high SNR regime

the roles of the two relays so that the channel is converted to a weak interference channel with interference gain of $1/a$, and the strength of the second hop is also changed accordingly.

For the decode-and-forward relaying, the DPC scheme and MAC scheme are both needed for the second hop. The combination of the two may sometimes outperform both of the individual schemes due to the time sharing effect. Generally however, DPC scheme dominates when b is small and MAC scheme dominates when b is large.

The comparison of decode-and-forward relaying and amplify-and-forward relaying showed that decode-and-forward relaying always has better performance except when a is close to b in the high SNR regime.

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